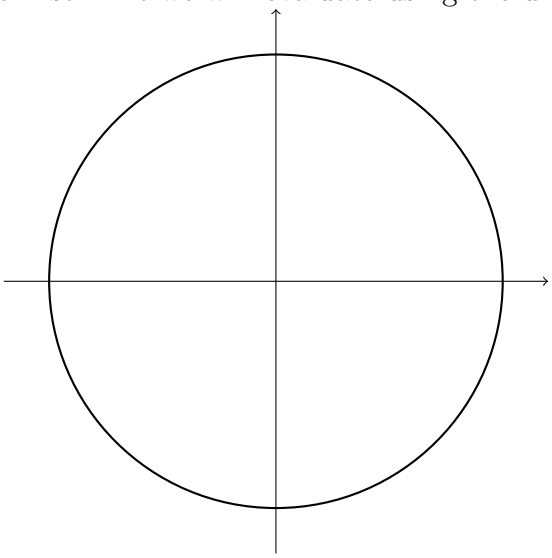
Section 3.5

Two Important Limits

The primary use for derivative rules is to directly compute derivatives rather than rely on the limit definition for every computation. We accomplish this by discovering specific limits, that once computed give us a general rule. For trigonometric functions these two limits are

$$\lim_{h\to 0}\frac{\sin h}{h} \quad \text{and} \quad \lim_{h\to 0}\frac{\cos h - 1}{h}.$$

The first limit we will evaluate using the unit circle:



The second limit we use a trig-identity.

Now let's differentiate sine and cosine.

$$\frac{d}{dx}\big(\sin x\big) =$$

$$\frac{d}{dx}\big(\cos x\big) =$$

Theorem 3.5.1 (Derivatives of Sine and Cosine)**.**

$$\frac{d}{dx}\big(\sin x\big) = \cos x \quad \text{and} \quad \frac{d}{dx}\big(\cos x\big) = -\sin x$$

Compare the graphs of sine and cosine.

Example 3.5.2. Calculate the derivatives of the following functions

1. $f(x) = (x^2 + 1)\sin x$

2. $g(x) = \cos^2 x$

3. $h(x) = e^{-x}\sin x$.

What about the other trig functions? Differentiate $\tan x$:

Theorem 3.5.3 (The Derivatives of Trigonometric Functions)**.**

$$\frac{d}{dx}\big(\sin x\big) = \cos x \qquad \frac{d}{dx}\big(\cos x\big) = -\sin x$$

$$\frac{d}{dx}\big(\tan x\big) = \sec^2 x \qquad \frac{d}{dx}\big(\cot x\big) = -\csc^2 x$$

$$\frac{d}{dx}\big(\sec x\big) = \sec x \tan x \qquad \frac{d}{dx}\big(\csc x\big) = -\csc x \cot x$$

Example 3.5.4. Let $y = \sec x$. Find $\frac{d^2y}{dx^2}$.

Example 3.5.5. Let $f(x) = \dfrac{1}{\sec x \csc x}$. Find $f'(x)$.